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J80-135

On the Determination of the Polytropic Specific Heat

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Nomenclature

c_n = polytropic specific heat
c_p = specific heat at constant pressure
c_v = specific heat at constant volume
k = ratio of specific heats c_p/c_v
m = mass
n = polytropic exponent
p = pressure
Q = transferred heat
S = entropy
T = temperature

Subscripts

1 = initial condition
2 = final condition

Introduction

DURING an isobaric process, transferred heat and changes in entropy are given in differential form by

$$dQ = mc_p dT \tag{1}$$

and

$$dS = mc_p dT/T \tag{2}$$

respectively. Accuracy permitting, their integration is usually performed for constant specific heat c_p or some acceptable mean value $\bar{c}_p$. Similarly, the corresponding integrated equations for an isometric process are

$$Q = mc_v (T_2 - T_1) \tag{3}$$

and

$$S_2 - S_1 = mc_v \ln T_2/T_1 \tag{4}$$

which differ from the integrated isobaric process equations only in the specific heat factor.

In the case of a polytropic process, the mathematical development leads to transferred heat and changes in entropy expressions of the form

$$dQ = mc_v \left(\frac{k-n}{1-n}\right) dT \tag{5}$$

and

$$dS = mc_v \left(\frac{k-n}{1-n}\right) \frac{dT}{T} \tag{6}$$

respectively, where $c_v(k-n)/(1-n)$ is customarily called the polytropic specific heat, denoted by c_n, so that at least in form the equations for both transferred heat and entropy changes are similar to the preceding cases, i.e.,

$$dQ = mc_n dT \tag{5a}$$

and

$$dS = mc_n dT/T \tag{6a}$$

Here, as in the preceding cases, the desirability of a constant c_n or an acceptable mean value $\bar{c}_n$ for integration purposes is obvious.

However, while the literature abounds with tabular and graphical data for c_p and c_v, such is not the case for c_n, and while the applicable literature routinely refers to the polytropic process as the most general of the "labeled" processes—substitution of the appropriate values for n will indeed yield these processes—as special cases of the polytropic process the effect of variable n on the polytropic specific heat itself, in general, and the significance of small variations of n with respect to c_n, in specific, are neither shown nor discussed. The latter point is of no small interest since rarely, if ever, is n known exactly throughout an entire process.

Analysis

In the following, it will be shown that the expression established for the polytropic specific heat, i.e., $c_n = c_v(k-n)/1-n)$, represents a well-defined geometric figure which, once recognized, admits of rapid determination of c_n as well as the prediction of effects of uncertainties in n for any gas whose specific heat ratio k is known.

When c_n is rewritten in the nondimensionalized form $c_n/c_v = (k-n)/(1-n)$ and its functional dependence is indicated by setting $f(n) = c_n/c_v$, the relation $f(n) = (k-n)/(1-n)$ may be examined from a purely mathematical point of view. Treating n and $f(n)$ as the axes of a Cartesian coordinate system, it becomes immediately apparent that this relation exhibits two asymptotes, one at $n = 1$ and the other at $f(n) = 1$.

The presence of these rectangular asymptotes with common point of intersection at (1,1) seems to indicate that the graphical representation of the polytropic specific heat equation is a translated and clockwise-rotated hyperbola with center at $(n, f(n)) = (1,1)$ and an angle of rotation equal to 45 deg.

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It is not immediately apparent that $f(n)=(k-n)/(1-n)$ represents a hyperbola; however, transforming this relation via the standard transformation-of-coordinates equations into a second, primed-coordinate system with origin at the center of the hyperbola and rotated as stated above will yield the transformed relation

$$\frac{n'^2}{2(k-1)}-\frac{f(n)'^2}{2(k-1)}=1 \tag{7}$$

which, indeed, is the standard equation of an equilateral hyperbola with semitransverse and semiconjugate axes equal to $\sqrt{2(k-1)}$ and all other hyperbola characteristics now readily determinable.

However, and more importantly, from the thermodynamics point of view: Knowing that the polytropic specific heat equation represents a hyperbola, an overview of the complete range of c_n values now becomes readily available.

Application of Theory

Since certain actual expansion and compression processes of fluids may be described by a polytropic pressure-volume relationship as is the case in reciprocating compressors, for example, a numerical application of the presented theory to such a system is shown here in order to highlight the insight gained from the polytropic specific heat hyperbola.

Consider a reciprocating air compressor with inlet temperature and pressure of 294 K (530°R) and 1.01×10^5 N/m^2 (14.7 psia), respectively, having a compression ratio of 5, and a polytropic exponent $n=1.35$.

For an ideal gas, substitution of the given conditions into the polytropic pressure-temperature relation

$$T_2/T_1=(p_2/p_1)^{(n-1)/n} \tag{8}$$

yields a discharge temperature $T_2=447$ K (804°R) with $c_n=-0.0245$ kcal/kg·K (-0.005 Btu/lb·°R) directly readable from an enlarged Fig. 1, resulting in transferred heat $Q/m=-3.73$ kcal/kg (-6.71 Btu/lb). Allowing for a possible 10% uncertainty in n, the hyperbola indicates the more serious effects in the direction of decreasing n. Therefore, choosing $n=1.22$ results in a discharge temperature $T_2=394$ K (709°R), a $c_n=-0.140$ kcal/kg·K (-0.029 Btu/lb.°R), and $Q/m=-13.9$ kcal/kg (-25.0 Btu/lb), representing a 273% change in transferred heat.

To explore the region of lesser sensitivity, consider a polytropic process with $n=3$, initial temperature, and initial and final pressures as stated previously.

For this n, the polytropic relations yield $T_2=860$ K (1550°R), $c_n=0.137$ kcal/kg·K (0.028 Btu/lb·°R), and a resultant unit mass heat transfer of $Q/m=77.5$ kcal/kg (140 Btu/lb). Again, allowing for a 10% uncertainty in n, the hyperbola predicts a relatively small change in the transferred heat in the direction of increasing n. Indeed, $n=3.3$ yields $T_2=904$ K (1630°R), $c_n=0.142$ kcal/kg·K (0.029 Btu/lb·°R), and a unit mass heat transfer of $Q/m=86.3$ kcal/kg (155 Btu/lb) which, by comparison with $n=3$ values, represents a change of 11%.

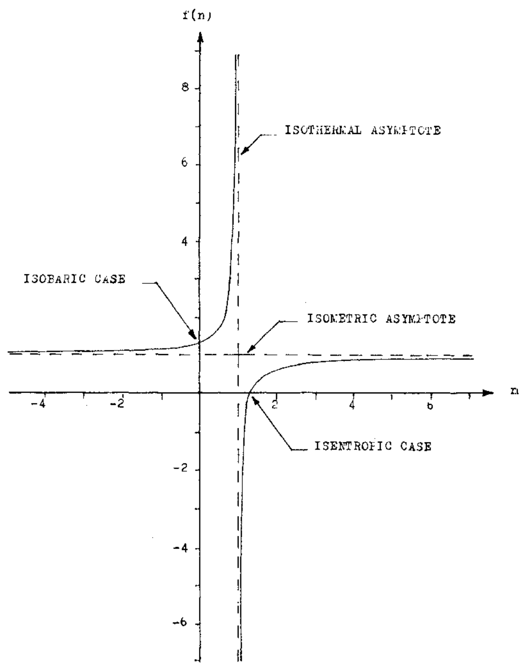

Fig. 1 Nondimensionalized polytropic specific heat hyperbola, $k=1.4$.

Discussion and Conclusions

Figure 1 shows the nondimensionalized polytropic specific heat as a function of the polytropic exponent for $k=1.4$, with the locations of the special-case processes identifed on the polytropic specific heat hyperbola.

Figure 1 also shows that the mathematical expression for the polytropic specific heat represents a translated and rotated equilateral hyperbola and, therefore, on the basis of just a few strategically determined data points it is possible to map out the entire range of the polytropic specific heat values, to discern by inspection the effects of small variations in n, and to determine if and where mean values are admissible.

The numerical examples presented show, as predicted by the polytropic specific heat hyperbola, that in the region of rapidly varying c_n, small changes in n, as they may occur in actual processes, produce significant changes in transferred heat, while in the region of lesser sensitivity the same variation in n produces correspondingly small changes in transferred heat.

While the ease with which the desired information may be obtained is obvious, perhaps the most significant insight offered by this graph is the realization of the effects of small fluctuations in n on c_n. When n is near 1 on either side, even minor deviations from its given value will produce substantial changes in the value of c_n, and in view of the fact that many actual fluid processes are governed by a polytropic pressure-volume relationship, to be able to identify critical regions in advance should be of decided advantage.

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Solar Flux Incident on an Orbiting Surface after Reflection from a Planet

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Introduction

FOR spacecraft orbiting Earth or other planets, the incoming fluxes due to direct insolation, planetary (infrared) irradiation, and solar radiation reflected from the

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